Abilities and Success

David Boylan*

Rutgers University, New Brunswick, New Brunswick, NJ, USA
db1082@philosophy.rutgers.edu

Does success entail ability? Call the principle that it does *Success*:

Success. S ϕ's $\models$ S can ϕ

When we focus on successful action, *Success* is compelling: when someone succeeds in something, like sinking a putt or surfing a wave, one is forced to concede they were able to do that. This is what *Success* would lead us to expect. But when success is not yet assured, the lesson seems different. When said before the fact, the claim that I can surf that wave is strong — it says that surfing that wave is within my control. This intuition drives against *Success*. Just doing something does not demonstrate it is within my control: flukes do happen. So, if the control intuition is right, success should *not* demonstrate ability.

First I try to make the above tension precise. I argue that the appeal of *Success* is connected to two plausible and related principles: that *past* success entails *past* ability, which I call *Past Success*; and that *cannot* seems to entail *will not*, which I call *Can't-entails-won't*. But, on the other, I show we can find counterexamples to *Success* in cases of inexact ability discussed by Kenny [1976]. To explain these data, I maintain we must connect the truth of ability claims to the facts about what our options *settle* and what they leave *open*, in the sense familiar from the literature on future contingents. I do this within a kind of conditional analysis of ability ascriptions. I first define an operator $\mathcal{W}$ with features attributed to 'will' in the literature on future contingents. In particular, $\mathcal{W}\phi$ is indeterminate in truth-value, when ϕ is unsettled. Building on previous joint work in Mandelkern et al. [2016], I state my conditional analysis in terms of $\mathcal{W}$-conditionals: on my view, ⌜S can ϕ⌝ says, roughly, there's some action available to S such that if S does it, then $\mathcal{W}$(S ϕ's) is true. By thus building a connection between unsettledness and indeterminacy into ability claims, my conditional account of abilities reconciles the motivations for *Success* with its counterexamples.

1 The Status of *Success*

Two facts are easy to explain, if *Success* is valid and hard to explain otherwise. To appreciate the first, let's focus on relatively mundane cases inspired by Kenny [1976]'s discussion of abilities:

> **Fluky Dartboard.** I am a terrible dartplayer. I struggle to even hit the board whenever I take a shot. However, I take my shot and I flukily hit the bullseye.

Once I have taken the shot and hit the bullseye, I can compellingly argue:

(1) I hit the bullseye on that throw.
So, I was able to hit the bullseye on that throw.

If you know that I have been successful, you must concede I was able to.

*Thanks to Fabrizio Cariani, Melissa Fusco, Jeff King, Justin Khoo, Matthew Mandelkern, Milo Phillips-Brown, Robert Stalnaker, and, especially, Ginger Schultheis.

This is our first point in favour of *Success*: *past* success feels like it entails past ability. Where $\rightsquigarrow$ denotes a felt entailment, we have:[1]

Past Success. $PAST$ (S ϕ'd) $\rightsquigarrow$ $PAST$ (S can ϕ)

This seems like strong motivation for *Success* itself. How could *Past Success* be valid, if *Success* is not? After all, *I* haven't changed since I hit the bullseye. My abilities are what they were some moments ago.

The second fact is that it sounds incoherent to affirm or leave open the premise of *Success*, but deny the conclusion. Consider:

(2) # I can't hit the bullseye on this shot, but I will.

(3) # I can't hit the bullseye on this shot, but I might.

This is our second point in favour of *Success*. In general, it seems that *can't* entails *won't*:

Can't-entails-won't. $\neg$(S can ϕ) $\rightsquigarrow$ $\neg$(S ϕ's)

And *Can't-entails-won't* is just the contrapositive of *Success*.

That is the case for *Success*, as I see it. But at the end of the day, I think it cannot be valid. Recall the control intuition: if I say something like

(4) I can hit the bullseye on this throw.

I say something quite strong. (4) is not verified by the small chance of me hitting the dartboard. We can leverage this intuition to find counterexamples to *Success*. Take the following variation on the dartboard case inspired by Kenny [1976]:

> **Unreliable Dartboard.** I am a fairly bad dartplayer. I regularly hit the bottom half when I aim for the top; and vice versa. But I never miss the board entirely.

I am about to take a shot. I am skilled enough to know I will hit the board; so I know that I will either hit the top half of the board or the bottom half of the board.[2] But it does not seem that I should ascribe myself either of the following abilities here:

(5) I can hit the top on this throw.

(6) I can hit the bottom on this throw.

Even the disjunction does not seem true:

(7) I can hit the top of the board or I can hit the bottom of the board.

So, in advance of the shot, even if I know I will hit the board somewhere, I do not have the ability to hit the top or the ability to hit the bottom.

---

[1] This is something like the converse of the actuality entailments discussed by Bhatt [1999] and Hacquard [2006].

[2] Note that here and throughout I assume the following principle:

Will Excluded Middle. Will $\phi \vee$ Will $\neg\phi$

This principle is widely taken to be extremely plausible. And, even given a view that *denies* Will Excluded Middle, no existing view of abilities will deliver both the validity of *Past Success* and *Can't-entails-won't* and the failure of $\vee$-*Success* below.

But I would be predicted to have one ability or the other if *Success* were valid. For I will hit either the top or the bottom of the dartboard. In the first case, *Success* says I'm able to hit the top, in the second that I'm able to hit the bottom. In either case, then, (7) is true, if *Success* is valid.

The consequences of *Success* become more absurd as the disjunctions get longer. Suppose we divide the dartboard into a million tiny, numbered regions. I can see that I will hit (at least) one of these regions because I know I will hit the dartboard, yielding:

(8) I will hit region 1 or 2 or 3 or ... or 1,000,000 on this throw.

If *Success* is valid, then the following incredible claim is true:

(9) I can hit region 1 on this throw or I can hit region 2 on this throw or ... or I can hit region 1,000,000 on this throw.

And, to put an even finer point on it, notice that each disjunct of (9) entails:

(10) There is a certain point that I can hit on this throw.

Success predicts that (10) should just be a truism here. In fact, (10) would be an incredible boast for me to make in **Unreliable Dartboard**.

So you can be sure that you will make true some disjunction, while failing to have the ability to make true either disjunct. This gives us the failure of an instance of *Success*:

$\vee$- *Success.* S ϕ's $\vee$ ψ's $\not\to$ S can ϕ $\vee$ S can ψ

Predicting this combination of data is a serious challenge. On standard theories, *Past Success* and *Can't-entails-won't* are both equivalent to *Success*. On the modal analysis of ability ascriptions, defended by Hilpinen [1969], Lewis [1976], Kratzer [1977] and Kratzer [1981a], reflexivity of the modal domain is necessary and sufficient for all three conditions. Brown [1988], Horty and Belnap [1995] and Horty [2001] all defend a view equivalent to the following:

Boxy Analysis. $[\![\text{S can } \phi]\!]^w = 1$ iff $\exists p \in \mathcal{A}(w) : \forall w' \in p : [\![\text{S } \phi\text{'s}]\!]^{w'} = 1$.

But again, all three inference patterns are equivalent: each is characterised by the condition that $\{w\} \in \mathcal{A}(w)$. Since we have seen that they are not equivalent, a new semantics is needed. (This also illustrates how the puzzle here goes beyond Kenny [1976]'s puzzle about disjunction: even views predict ability fails to distribute over disjunction fail to solve my puzzle.)

2 Future Contingents and $\mathcal{W}$

I will explain these data by connecting ability claims to *future contingents.* Future contingents have been argued to have three special properties. As we will see, each property mirrors a property of ability. Our assessments of 'will'-claims, it has been argued, involve a kind of temporal asymmetry. Before the fact, the future seems open and 'will'-claims seem unsettled; but after the fact, we seem happy to talk as if they were settled all along. 'Will' is also scopeless with respect to negation: $\ulcorner\neg$Will $\phi\urcorner$ seems equivalent to $\ulcorner$Will $\neg\phi\urcorner$.

Here I introduce a modal operator $\mathcal{W}$ that captures this behaviour. $\mathcal{W}$ is a selection modal, in the sense of Cariani and Santorio [2018]: $\ulcorner\mathcal{W}\phi\urcorner$ says that ϕ is true in the closest world to actuality. But crucially, on my theory, the closest world leaves various facts unsettled. This ensures $\mathcal{W}$ has the right properties for giving the semantics of ability modals.

2.1 Features of Future Contingents

Let's first take the temporal asymmetry in assessing future contingents.

Recall Aristotle's famous case of the sea battle. On Monday, it is not yet settled whether there will be a battle or not on Tuesday: a capricious ruler decides by flipping a fair coin this evening. I make the following prediction:

(11) There will be a sea battle on Tuesday.

There is a long tradition of thinking that because the future is unsettled, sentences like (11) must be *indeterminate* in truth-value.[3] Things could go either way, depending on how the coin lands. If the coin comes up heads, there will be a battle; if not then not. But the outcome of the toss is not settled; and so whether there will be a sea battle tomorrow is unsettled too.

Now suppose the sea battle does take place on Tuesday. When I look back on my earlier prediction, what should I think? It seems I can say either of the following:

(12) There would in fact be a sea battle on Tuesday.

(13) I said there would be a sea battle on Tuesday; and indeed there would be.

'Would' is generally regarded to be the past tense of 'will'.[4] But then it is surprising that we can say either of (12) or (13): if my assertion was indeterminate when I said it, why do I now say that it was true?[5]

These are the first two properties I want $\mathcal{W}$ to have. I want $\mathcal{W}$ to obey two inference patterns. Where $\Diamond\phi$ says that ϕ is circumstantially possible and $\nabla\phi$ says that ϕ is indeterminate:

Openness. $\Diamond\phi, \Diamond\neg\phi \rightsquigarrow \nabla\, \mathcal{W}\phi$

Past Settledness. $PAST\ \phi \rightsquigarrow PAST\ \mathcal{W}\phi$

This will eventually allow me to validate *Past Success* without validating *Success.*

The third feature is the way that 'will' interacts with negation. 'Will' does not give rise to any scope distinctions with respect to negation. Take a predicate like 'is absent' that includes a negation as part of its meaning. Consider the following example:

(14) I doubt that John will be present.

This says that I think it is *not* the case that John will be present. But it quite clearly entails

(15) I think that John will be absent.

In general, saying that it is not the case that ϕ will happen just is to say that $\neg\phi$ will happen.[6] That is, $\mathcal{W}$ should be *scopeless* with respect to negation, as Cariani and Santorio [2018] put it:

Scopelessness. $\neg\mathcal{W}\phi \leftrightsquigarrow \mathcal{W}\neg\phi$

Scopelessness will secure *Can't-entails-won't.*

[3] See, among others, Aristotle, *De Interpretatione*, Lukasiewicz [1920], Lukasiewicz [1951].
[4] See Abusch [1997] and Condoravdi [2002].
[5] This point was first raised in Prior [1976] and later repeated in MacFarlane [2003].
[6] This point has been recognised at least since Thomason [1970].

2.2 Semantics for $\mathcal{W}$

Following Cariani and Santorio [2018], I say that $\ulcorner \mathcal{W}\phi \urcorner$ is true iff ϕ is true in the *closest world* to the actual world. Unlike Cariani and Santorio, however, I say that the closest world can be unsettled in various respects. As shown by Prior [1967], Thomason [1970] and Thomason [1984], a world that is unsettled past a certain time can be represented using a *set* of worlds which agree in all (relevant) matters of fact up to that time, but diverge afterwards; I'll call such a set an *unsettled world*. In the sea battle case, we represent the earlier, indeterminate state of the world with a set of worlds agreeing on all (relevant) matters of fact up until today and then diverging on whether a sea battle occurs tomorrow. A proposition is true at an unsettled world iff it is true at all worlds in that set; it is false iff it is false at all worlds in that set; and indeterminate, otherwise.

To state the lexical entry for $\mathcal{W}$, let's first say how unsettled worlds get into the semantics. I add an unsettled world, $\mathcal{I}$, to the index of the semantic evaluation function $[\![\cdot]\!]$. I assume that the unsettled world is supplied to the semantics by context: we form the unsettled world of the context $\mathcal{I}_c$ by taking the set of worlds that are duplicates of the (determinate) context world up until the context time:

Unsettled World. $\mathcal{I}_c = \{w \mid w \text{ is identical to } w_c \text{ up until } t_c\}$

Now let's consider how to model closeness. Following Stalnaker [1968], I use a selection function to supply the closest worlds. A Stalnakerian selection function s takes a world w and a proposition $\mathbf{A}$ and returns the closest world to w where $\mathbf{A}$ is true. My selection functions take an unsettled world as input and can also return an unsettled world as output: $s(\mathcal{I}, \mathbf{A})$ picks out the closest (possibly unsettled) world to $\mathcal{I}$ which settles that $\mathbf{A}$ is true.

What if we want the selection function to give us the closest world to $\mathcal{I}$ simpliciter? We simply let the other argument be the tautology $\top$. I say that $\top$ is supplied by a *modal base*, a function f from a world and a time to a set of worlds.[7] I assume that f is supplied by the index and that f does not include any information by itself:

Modal Bases. $f_c(w, t) = W$

Thus, to find the closest world simpliciter to the unsettled world of the context, $\mathcal{I}_c$, we find $s(\mathcal{I}_c, f_c(w_c, t_c))$.

I will make a structural assumption about closeness, which I call *Overlap*:[8]

Overlap. If $\mathbf{A} \cap \mathbf{B} \neq \emptyset$, then $s(\mathbf{A}, \mathbf{B}) = \mathbf{A} \cap \mathbf{B}$.

Suppose we want the closest worlds to an unsettled world $\mathbf{B}$ where $\mathbf{A}$ happens and that $\mathbf{B}$ contains some $\mathbf{A}$-worlds. *Overlap* tells us that those closest worlds will be the $\mathbf{A}$-worlds in $\mathbf{B}$.

Overlap guarantees that $\mathcal{I}_c$ is the closest world to itself. Since $f_c(w_c, t_c)$ is the set of all worlds, $s(\mathcal{I}_c, f_c(w_c, t_c))$ must be $\mathcal{I}_c$. This allows us to ignore the modal base when $\mathcal{W}$ is unembedded or under past tense. (The modal base will be relevant, however, when we consider conditionals in the next section.)

Now let's state the semantics. To see if $\ulcorner \mathcal{W}\phi \urcorner$ is true, we find $s(\mathcal{I}, f(w, t))$, the closest world to the unsettled world where ϕ is true. If ϕ is true at $s(\mathcal{I}, f(w, t))$ (i.e. true throughout $\mathcal{I}$), $\ulcorner \mathcal{W}\phi \urcorner$ is true; if ϕ is false at $s(\mathcal{I}, f(w, t))$ (i.e. false throughout $\mathcal{I}$), $\ulcorner \mathcal{W}\phi \urcorner$ is false; but if ϕ is neither true nor false at $s(\mathcal{I}, f(w, t))$ (i.e. is true at some worlds in $\mathcal{I}$ but false at others),

[7] Cariani and Santorio also use a modal base, but do not assume it is empty.

[8] This is an analogue of the Strong Centering principle on Stalnaker selection functions, which says that w is always the closest world to itself.

$\ulcorner\mathcal{W}\phi\urcorner$ is indeterminate. Say that the value of $[\![\phi]\!]$ is *determinate* at a point just it is not indeterminate. Then we can make this precise as follows:

(16) a. $[\![\mathcal{W}\phi]\!]^{w,t,f,\mathcal{I}}$ is determinate only if either
(i) $s(\mathcal{I}, f(w,t)) \subseteq [\![\phi]\!]^{t,f,\mathcal{I}}$ or
(ii) $s(\mathcal{I}, f(w,t)) \subseteq [\![\neg\phi]\!]^{t,f,\mathcal{I}}$
b. If determinate, $[\![\mathcal{W}\phi]\!]^{w,t,f,\mathcal{I}} = 1$ iff $s(\mathcal{I}, f(w,t)) \subseteq [\![\phi]\!]^{t,f,\mathcal{I}}$

We also need to make explicit some background assumptions. I assume the following standard semantics for negation, the past and the indeterminacy operator.

(17) $[\![\neg\phi]\!]^{w,t,f,\mathcal{I}} = 1$ iff $[\![\phi]\!]^{w,t,f,\mathcal{I}} = 0$
$[\![\neg\phi]\!]^{w,t,f,\mathcal{I}} = 0$ iff $[\![\phi]\!]^{w,t,f,\mathcal{I}} = 1$.
$[\![\neg\phi]\!]^{w,t,f,\mathcal{I}} = \#$ iff $[\![\phi]\!]^{w,t,f,\mathcal{I}} = \#$.

(18) $[\![PAST\ \phi]\!]^{w,t,f,\mathcal{I}} = 1$ iff $\exists t' < t : [\![\phi]\!]^{w,t',f,\mathcal{I}} = 1$

(19) $[\![\nabla\phi]\!]^{w,t,f,\mathcal{I}} = 1$, if $[\![\phi]\!]^{w,t,f,\mathcal{I}} = \#$;
$[\![\nabla\phi]\!]^{w,t,f,\mathcal{I}} = 0$, otherwise.

I assume a standard semantics for the circumstantial modal and make the standard assumption about its accessibility relation $C(w,t)$:

(20) $[\![\Diamond\ \phi]\!]^{w,t,f,\mathcal{I}} = 1$ iff some $w' \in C(w,t)$: $[\![\phi]\!]^{w',t,f,\mathcal{I}} = 1$

Circumstantial. $C(w,t) = \{w' \mid w'$ is identical to w up until $t\}$

Finally, I assume entailment is preservation of truth at a context:

Truth at a context. $[\![\phi]\!]^c = [\![\phi]\!]^{w_c,t_c,f_c,\mathcal{I}_c}$

Diagonal validity. $\phi_1, ..., \phi_n \models_c \psi$ iff whenever $[\![\phi_1]\!]^c = ... = [\![\phi_n]\!]^c = 1$ then $[\![\psi]\!]^c = 1$.

2.3 Delivering Openness, Past Settledness and Asymmetry

This package delivers our three features of future contingents:

Fact 1. $\Diamond\phi$, $\Diamond\neg\phi \models_c \nabla(\mathcal{W}\phi)$

Proof. Suppose $[\![\Diamond\phi]\!]^c = [\![\Diamond\neg\phi]\!]^c = 1$. Then $C(w_c,t_c) \not\subseteq [\![\phi]\!]^{t_c,f_c,\mathcal{I}_c}$ and $C(w_c,t_c) \not\subseteq [\![\neg\phi]\!]^{t_c,f_c,\mathcal{I}_c}$. By *Unsettled World*, $\mathcal{I}_c = C(w_c,t_c)$, so $\mathcal{I}_c \not\subseteq [\![\phi]\!]^{t_c,f_c,\mathcal{I}_c}$ and $\mathcal{I}_c \not\subseteq [\![\neg\phi]\!]^{t_c,f_c,\mathcal{I}_c}$. By *Overlap* and *Modal Bases*, $s(\mathcal{I}_c, f_c(w_c,t_c)) = \mathcal{I}_c$. Since then $s(\mathcal{I}_c, f_c(w_c,t_c)) \not\subseteq [\![\phi]\!]^{t_c,f_c,\mathcal{I}_c}$ and $s(\mathcal{I}_c, f_c(w_c,t_c)) \not\subseteq [\![\neg\phi]\!]^{t_c,f_c,\mathcal{I}_c}$, $[\![\mathcal{W}\phi]\!]^{w_c,t_c,f_c,\mathcal{I}_c} = \#$; so $[\![\nabla(\mathcal{W}\phi)]\!]^{w_c,t_c,f_c,\mathcal{I}_c} = 1$.

Fact 2. Where ϕ is non-modal, $PAST\ \phi \models_c PAST(\mathcal{W}\phi)$

Proof. Suppose $[\![PAST\ \phi]\!]^c = 1$. Then there's some t' earlier than t_c such that $[\![\phi]\!]^{w_c,t',f_c,\mathcal{I}_c} = 1$. By *Circumstantial* and *Unsettled World*, it follows that $\mathcal{I}_c \subseteq [\![\phi]\!]^{t',f_c,\mathcal{I}_c}$. By *Modal Bases* and *Overlap*, we have $s(\mathcal{I}_c, f(w,t')) = \mathcal{I}_c$. So there is some $t' < t_c$, such that $s(\mathcal{I}_c, f(w,t')) \subseteq [\![\phi]\!]^{t',f_c,\mathcal{I}_c}$. So $[\![PAST(\mathcal{W}\phi)]\!]^{w_c,t_c,f_c,\mathcal{I}_c} = 1$.

Fact 3. $\neg\mathcal{W}\phi \models_c \mathcal{W}\neg\phi$

Proof. Suppose $[\![\neg\mathcal{W}\phi]\!]^c = 1$. Since $[\![\neg\mathcal{W}\phi]\!]^c = 1$ is determinate, it must be that either i) $s(\mathcal{I}_c, f_c(w_c,t_c)) \subseteq [\![\phi]\!]^{t_c,f_c,\mathcal{I}_c}$ or ii) $s(\mathcal{I}_c, f_c(w_c,t_c)) \subseteq [\![\neg\phi]\!]^{t_c,f_c,\mathcal{I}_c}$. Case i) is inconsistent with our initial assumption, so case ii) holds. That suffices for $[\![\mathcal{W}\neg\phi]\!]^c = 1$

3 A Conditional Semantics for Abilities

Now that we have introduced $\mathcal{W}$, I can give my semantics for abilities. On my theory, ⌜S can ϕ⌝ is true iff, for some available action α, if S α's then $\mathcal{W}$(S ϕ's).[9] To build up to a precise statement, we will first say more about actions, conditionals and the projection of indeterminacy.

To simplify, propositions about actions stand in for actions. I represent the available actions using a function $\mathcal{A}$ that takes a world and a time and yields a set of propositions. $\mathcal{A}(w,t)$ is the set of actions available to the subject at w and t.

I make two important formal assumptions about actions. First, I assume the set of available actions is a *partition* of the circumstantially possible worlds:[10]

Partitionality. $\mathcal{A}(w,t)$ is a partition of $C(w,t)$, the circumstantially accessible worlds at w and t.

Importantly, since the actual world is always circumstantially accessible to itself, the actual world is always a member of an available action. Secondly, I add the assumption that if an action available in the past was performed, then it is settled that it was performed:

Action Time. If $t < t_c$, then if $\alpha \in \mathcal{A}(w_c,t)$ and $w_c \in \alpha$ then $\mathcal{I}_c \subseteq \alpha$.

To fully spell out my conditional analysis, we need to be clear on how the conditional works. I give it a *restrictor* semantics, a la Kratzer [1981b, 2012]: a conditional restricts the domain for a modal in the consequent to worlds where the antecedent is true.[11] More precisely, where $f^{\mathbf{A}}$ is the function such that $f^{\mathbf{A}}(w) = f(w) \cap \mathbf{A}$, the conditional has the following truth-conditions.

(21) $[\![\text{if } \phi,\ \psi]\!]^{w,t,f,\mathcal{I}} = 1$ iff $[\![\psi]\!]^{w,t,f^{[\![\phi]\!]^{t,f,\mathcal{I}}},\mathcal{I}} = 1$

When $\mathcal{W}$ is embedded in the consequent, a restrictor conditional restricts the modal base argument to the selection function. ⌜if ϕ, $\mathcal{W}\psi$⌝ is true iff the closest world to the actual unsettled world *where ϕ is true* is one where ψ is true:

(22) $[\![\text{if } \phi,\ \mathcal{W}\psi]\!]^{w,t,f,\mathcal{I}} = 1$ iff $[\![\mathcal{W}\psi]\!]^{w,t,f^{[\![\phi]\!]^{t,f,\mathcal{I}}},\mathcal{I}} = 1$
iff $s(\mathcal{I}, f^{[\![\phi]\!]^{t,f,\mathcal{I}}}(w,t)) \subseteq [\![\psi]\!]^{t,f^{[\![\phi]\!]^{t,f,\mathcal{I}}},\mathcal{I}} = 1$

Finally, we need to say how indeterminacy projects. I assume a Strong Kleene approach. On a Strong Kleene approach to disjunction, ⌜ϕ or ψ⌝ is determinate when we have enough information to determine a unique truth-value using the classical truth-table for 'or'. If at least one of ϕ and ψ is true, ⌜ϕ or ψ⌝ is true; if both are false, ⌜ϕ or ψ⌝ is false; in all remaining cases it is indeterminate. This idea carries over to existential quantifiers: an existentially quantified sentence is true if it has a true instance; it is false if it has only false instances; and indeterminate otherwise.

[9] As mentioned, this semantics builds on the account of Mandelkern et al. [2016].
[10] This is a standard move in the literature in deontic modals. See for instance Cariani [2013].
[11] Kratzer assumes bare conditionals involve a tacit epistemic 'must' in the consequent.

Now apply this to 'can'. I say that ⌜S can ϕ⌝ is true when, for some available α, ⌜If S α's, $\mathcal{W}$(S ϕ's)⌝ is true; ⌜S can ϕ⌝ is false when, for every available α, ⌜If S α's, $\mathcal{W}$(S ϕ's)⌝ is false; and is indeterminate, otherwise. Spelled out precisely:

(23) $[\![\text{S can } \phi]\!]^{w,t,f,\mathcal{I}}$ is determinate only if either
  a. there is some $\alpha \in \mathcal{A}(w,t)$ such that $[\![\mathcal{W}(\text{S } \phi\text{'s})]\!]^{w,t,f^{\alpha},\mathcal{I}} = 1$;
  b. or for all $\alpha \in \mathcal{A}(w,t)$, $[\![\mathcal{W}(\text{S } \phi\text{'s})]\!]^{w,t,f^{\alpha},\mathcal{I}} = 0$.

Putting this altogether, here is the full statement of the view:

(24) a. $[\![\text{S can } \phi]\!]^{w,t,f,\mathcal{I}}$ is determinate only if either
  (i) there is some $\alpha \in \mathcal{A}(w,t)$ such that $[\![\mathcal{W}(\text{S } \phi\text{'s})]\!]^{w,t,f^{\alpha},\mathcal{I}} = 1$;
  (ii) or for all $\alpha \in \mathcal{A}(w,t)$, $[\![\mathcal{W}(\text{S } \phi\text{'s})]\!]^{w,t,f^{\alpha},\mathcal{I}} = 0$.
  b. If determinate, $[\![\text{S can } \phi]\!]^{w,t,f,\mathcal{I}} = 1$ iff for some $\alpha \in \mathcal{A}(w,t) : [\![\mathcal{W}(\text{S } \phi\text{'s})]\!]^{w,t,f^{\alpha},\mathcal{I}} = 1$
  i.e. iff for some $\alpha \in \mathcal{A}(w,t) : s(\mathcal{I}, f(w,t) \cap \ \alpha) \subseteq [\![\text{S } \phi\text{'s}]\!]^{t,f^{\alpha},\mathcal{I}}$

Given our earlier entry for negation, this yields the following entry for 'cannot':

(25) a. $[\![\text{S cannot } \phi]\!]^{w,t,f,\mathcal{I}}$ is determinate only if either
  (i) there is some $\alpha \in \mathcal{A}(w,t)$ such that $[\![\mathcal{W}(\text{S } \phi\text{'s})]\!]^{w,t,f^{\alpha},\mathcal{I}} = 1$;
  (ii) or for all $\alpha \in \mathcal{A}(w,t)$, $[\![\mathcal{W}(\text{S } \phi\text{'s})]\!]^{w,t,f^{\alpha},\mathcal{I},\mathcal{A}} = 0$.
  b. If determinate, $[\![\text{S cannot } \phi]\!]^{w,t,f,\mathcal{I}} = 1$ iff for all $\alpha \in \mathcal{A}(w,t) : [\![\mathcal{W}(\text{S } \phi\text{'s})]\!]^{t,f^{\alpha},\mathcal{I}} \neq 1$
  i.e. iff for all $\alpha \in \mathcal{A}(w,t) : s(\mathcal{I}, f(w,t) \cap \alpha) \nsubseteq [\![\text{S } \phi\text{'s}]\!]^{t,f^{\alpha},\mathcal{I}}$
  i.e., given the determinacy conditions, iff for all $\alpha \in \mathcal{A}(w,t) : s(\mathcal{I}, f(w,t) \cap \alpha) \subseteq [\![\neg(\text{S } \phi\text{'s})]\!]^{t,f^{\alpha},\mathcal{I}}$

4 Predictions

I distilled the tension involving *Success* into three data points about ability modals:

Validity of *Past Success.* *PAST* (S ϕ's) $\rightsquigarrow$ *PAST* (S can ϕ)

Validity of *Can't-entails-won't.* $\neg$(S can ϕ) $\rightsquigarrow$ $\neg$(S ϕ's)

*In*validity of $\vee$- *Success.* S ϕ's $\vee$ S ψ's $\not\rightsquigarrow$ S can ϕ $\vee$ S can ψ

My semantics predicts all of these data. *Past Success* holds because if S actually did α and ϕ in the past, this suffices for the truth of ⌜PAST(if S α's, $\mathcal{W}$(S ϕ's))⌝. *Can't-entails-won't* is valid because ⌜S cannot ϕ⌝ says that for all available actions α, the closest world where S α's settles that S does not ϕ. Since some available action must always be performed, this ensures that S ϕ's. $\vee$-*Success* fails because S may end up ϕ-ing even though no available action *settles* whether S ϕ's.

More precisely, we can prove the following facts:

Fact 4. S ϕ's $\vee$ S ψ's $\not\models_c$ S can ϕ $\vee$ S can ψ.

Proof. Take S ϕ's and (S $\neg\phi$'s). Suppose:

1. $[\![\text{S } \phi\text{'s}]\!]^{w_c,t_c,f_c,\mathcal{I}_c} = 1$
2. $\mathcal{A}(w_c,t_c) = \{$*S tries to* ϕ, *S tries to* $\neg\phi\}$
3. *S tries to* ϕ and *S tries to* $\neg\phi$ are consistent with both $[\![\text{S } \phi\text{'s}]\!]^{t_c,f_c,\mathcal{I}_c}$ and $[\![(\text{S } \neg\phi\text{'s})]\!]^{t_c,f_c,\mathcal{I}_c}$

By 1, $[\![\text{S } \phi\text{'s} \vee \text{S } \neg\phi\text{'s}]\!]^{w_c,t_c,f_c,\mathcal{I}_c} = 1$. But the determinacy conditions for ⌜S can ϕ⌝ are not met: for no $\alpha \in \mathcal{A}_c(w_c,t_c)$: $\alpha \subseteq [\![\text{S } \phi\text{'s}]\!]^{t_c,f_c,\mathcal{I}_c}$ or $\alpha \subseteq [\![\text{S } \neg\phi\text{'s}]\!]^{t_c,f_c,\mathcal{I}_c}$. Similarly the determinacy conditions for ⌜S can $\neg\phi$⌝ are not met. So $[\![\text{S can } \phi]\!]^{w_c,t_c,f_c,\mathcal{I}_c} = [\![\text{S can } \neg\phi]\!]^{w_c,t_c,f_c,\mathcal{I}_c} = \#$. So S ϕ's $\vee$ S ψ's $\not\models_c$ S can ϕ $\vee$ S can ψ.

Fact 5. For non-modal ϕ, $PAST(\text{S } \phi\text{'s}) \models_c PAST(\text{S can } \phi)$

Proof. Suppose $[\![PAST(\text{S } \phi\text{'s})]\!]^{w_c,t_c,f_c,\mathcal{I}_c} = 1$. By our entry for *PAST*, there is some $t' < t$ such that $[\![\text{S } \phi\text{'s}]\!]^{w_c,t',f_c,\mathcal{I}_c} = 1$. By *Circumstantial*, that $C(w,t) \subseteq [\![\text{S } \phi\text{'s}]\!]^{t',f_c,\mathcal{I}_c}$; and by *Unsettled World*, $\mathcal{I}_c \subseteq [\![\text{S } \phi\text{'s}]\!]^{t',f_c,\mathcal{I}_c}$. Now by *Partition*, we know that there is some $\alpha \in \mathcal{A}_c(w_c,t',)$ such that $w_c \in \alpha$. *Action Time* gives us that $\mathcal{I}_c \subseteq \alpha$.

Now we can show that $s(\mathcal{I}_c, f_c(w_c,t') \cap \alpha) \subseteq [\![\text{S } \phi\text{'s}]\!]^{t',f_c,\mathcal{I}_c}$. By *Modal Bases*, $(f_c(w_c,t') \cap \alpha) = \alpha$. So $\mathcal{I}_c \subseteq f_c(w_c,t') \cap \alpha$. So, by *Overlap*, we have that $s(\mathcal{I}_c, f_c(w_c,t') \cap \alpha) = \mathcal{I}_c$. But we already know that $\mathcal{I}_c \subseteq [\![\text{S } \phi\text{'s}]\!]^{t',f_c,\mathcal{I}_c}$. So $\exists\alpha \in \mathcal{A}_c(w_c,t') : s(\mathcal{I}_c, f_c(w_c,t') \cap \alpha) \subseteq [\![\text{S } \phi\text{'s}]\!]^{t',f_c,\mathcal{I}_c}$. Since ϕ is non-modal, $[\![\text{S } \phi\text{'s}]\!]^{t',f_c,\mathcal{I}_c} = [\![\text{S } \phi\text{'s}]\!]^{t',f_c^\alpha,\mathcal{I}_c}$; and so for some $t' < t_c$, $[\![\text{S can } \phi]\!]^{w_c,t',f_c,\mathcal{I}_c} = 1$. So $[\![PAST(\text{S can } \phi)]\!]^c = 1$.

Fact 6. When ϕ is non-modal, S cannot $\phi \models_c \neg(\text{S } \phi\text{'s})$

Proof. Suppose $[\![\text{S cannot } \phi]\!]^{w_c,t_c,f_c,\mathcal{I}_c} = 1$. Since it is determinate, either i) there is some $\alpha \in \mathcal{A}_c(w_c,t_c)$ such that $[\![\mathcal{W}(\text{S } \phi\text{'s})]\!]^{w_c,t_c,f_c^\alpha,\mathcal{I}_c} = 1$; or ii) for all $\alpha \in \mathcal{A}_c(w_c,t_c)$, $[\![\mathcal{W}(\text{S } \phi\text{'s})]\!]^{w_c,t_c,f_c^\alpha,\mathcal{I}_c} = 0$. Since $[\![\text{S cannot } \phi]\!]^{w_c,t_c,f_c,\mathcal{I}_c} = 1$, i) cannot hold. ii) and the determinacy conditions for $\mathcal{W}$ gives us that for all $\alpha \in \mathcal{A}_c(w_c,t_c)$, $[\![\mathcal{W}\neg(\text{S } \phi\text{'s})]\!]^{w_c,t_c,f_c+\alpha,\mathcal{I}_c} = 1$. In other words, for all $\alpha \in \mathcal{A}_c(w_c,t_c)$, $s(\mathcal{I}_c, f_c(w_c,t_c) \cap \alpha) \subseteq [\![\neg(\text{S } \phi\text{'s})]\!]^{t_c,f_c,\mathcal{I}_c}$.

By *Partitionality*, *Circumstantial* and *Unsettled World*, there's some $\alpha \in \mathcal{A}_c(w_c,t_c)$ such that $w_c \in \alpha$. This, together with *Modal Bases*, ensures $w_c \in f^\alpha(w_c,t_c)$. By *Circumstantial* and *Unsettled World*, we know $w_c \in \mathcal{I}_c$. By *Overlap*, we then know $w_c \in s(\mathcal{I}_c, f_c^\alpha(w_c,t_c))$. But then, since $\forall\alpha \in \mathcal{A}_c(w_c,t_c)$, $s(\mathcal{I}_c, f_c(w_c,t_c) \cap \alpha) \subseteq [\![\neg(\text{S } \phi\text{'s})]\!]^{t_c,f_c^\alpha,\mathcal{I}_c}$, $w_c \in [\![\neg(\text{S } \phi\text{'s})]\!]^{t_c,f_c^\alpha,\mathcal{I}_c}$, i.e. $[\![\neg(\text{S } \phi\text{'s})]\!]^{w_c,t_c,f_c^\alpha,\mathcal{I}_c} = 1$. When ϕ is non-modal, $[\![\neg(\text{S } \phi\text{'s})]\!]^{w_c,t_c,f_c^\alpha,\mathcal{I}_c} = 1$ iff $[\![\neg(\text{S } \phi\text{'s})]\!]^{w_c,t_c,f_c,\mathcal{I}_c} = 1$. So $[\![\neg(\text{S } \phi\text{'s})]\!]^{w_c,t_c,f_c,\mathcal{I}_c} = 1$.

References

Dorit Abusch. Sequence of tense and temporal de re. *Linguistics and Philosophy*, 20(1):1–50, 1997.

Rajesh Bhatt. Ability Modals and Their Actuality Entailments. In K. Shahin, S. Blake, and E.-S. Kim, editors, *WCCFL 17 Proceedings*, volume 17, pages 74–87, 1999.

Mark A. Brown. On the Logic of Ability. *Journal of Philosophical Logic*, 17:1–26, 1988.

Fabrizio Cariani. 'Ought' and Resolution Semantics. *Nous*, 47(3):534–558, 2013.

Fabrizio Cariani and Paolo Santorio. *Will* done better: Selection semantics, future credence, and indeterminacy. *Mind*, 127(505):129–165, 2018.

Cleo Condoravdi. Temporal interpretations of modals. In David Beaver, Stefan Kaufman, Brady Clark, and Luis Casillas, editors, *The Construction of Meaning*, pages 59–88. CSLI Publications, 2002.

Valentine Hacquard. *Aspects of Modality*. PhD thesis, MIT, 2006.

Risto Hilpinen. An Analysis of Relativised Modalities. In J.W. Davis, D.J. Hockney, and W.K. Wilson, editors, *Philosophical Logic*, volume 20. Springer, 1969.

John F. Horty. *Agency and Deontic Logic*. Oxford University Press, 2001.

John F. Horty and Nuel Belnap. The deliberative stit: A study of action, omission, ability, and obligation. *Journal of Philosophical Logic*, 24(6):583–644, 1995.

Anthony Kenny. Human Abilities and Dynamic Modalities. In J. Manninen and R. Tuomela, editors, *Essays on Explanation and Understanding*, pages 209–232. D. Reidel, 1976.

Angelika Kratzer. What 'Must' and 'Can' Must and Can Mean. *Linguistics and Philosophy*, 1 (337-355), 1977.

Angelika Kratzer. The Notional Category of Modality. In H. Eikmeyer and H. Rieser, editors, *Words, Worlds, and Contexts: New Approaches in Word Semantics*. de Gruyter, 1981a.

Angelika Kratzer. The notional category of modality. In Hans-Jürgen Eikmeyer and Hannes Reiser, editors, *Words, Worlds and Contexts*, pages 38–74. Walter de Gruyter and Co., 1981b.

Angelika Kratzer. Conditionals. In *Modals and Conditionals*, chapter 4. Oxford University Press, 2012.

David Lewis. The Paradoxes of Time Travel. *American Philosophical Quarterly*, (145-152), 1976.

Jan Lukasiewicz. On three-valued logic. *Ruch filozoficzny*, 5(170-171), 1920.

Jan Lukasiewicz. *Aristotle's Syllogistic From the Standpoint of Modern Formal Logic*. Garland, 1951.

John MacFarlane. Future contingents and relative truth. *Philosophical Quarterly*, 53(212): 321–336, 2003. doi: 10.1111/1467-9213.00315.

Matthew Mandelkern, Ginger Schultheis, and David Boylan. Agentive modals. To appear, *The Philosophical Review*, 2016.

A.N. Prior. It was to be. In Peter Geach and Anthony Kenny, editors, *Papers in Semantics and Ethics*, pages 97–108. Duckworth, 1976.

Arthur Prior. *Past, Present and Future*. Clarendon Press, Oxford, 1967.

Robert C. Stalnaker. A theory of conditionals. *American Philosophical Quarterly*, pages 98–112, 1968.

Richmond H. Thomason. Indeterminist time and truth-value gaps. *Theoria*, 36(3):264–281, 1970. doi: 10.1111/j.1755-2567.1970.tb00427.x.

Richmond H Thomason. Combinations of tense and modality. In D Gabbay and F Guenthner, editors, *Handbook of Philosophical Logic: Extensions of Classical Logic*. Reidel, 1984.